Math 1320
C. Mundy-Castle
Spring '13

NAME KEY

FINAL EXAM

Tuesday, December 10th, 2013

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#3	/10
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#5	/10
#6	/10
#7	/10
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#9	/10
#10	/10
Total	/100

You may use all of your previous formula sheets (or a new formula sheet) on this exam. **Show all of your work to receive full/partial credit!**

1) Given $f(x) = 6 - 4x - 3x^2$, find

a) $f(-1)$ $= 6 - 4(-1) - 3(-1)^2 = 6 + 4 - 3 = 7$

b) $f(w) = 6 - 4w - 3w^2$

c) $f(x+h) = 6 - 4(x+h) - 3(x+h)^2$

$= 6 - 4x - 4h - 3(x^2 + 2xh + h^2)$

$= 6 - 4x - 4h - 3x^2 - 6xh - 3h^2$

2) A battery manufacturer can produce 5,000 batteries per day at a total cost of $2,000 and 11,000 batteries at a total cost of $3,200. Find the manufacturer's daily fixed costs and marginal cost per battery.

$$m = \frac{3200 - 2000}{11000 - 5000} = \frac{1200}{6000} = 0.2$$

$C(x) = 0.2x + b$

$2000 = 0.2(5000) + b$

$2000 = 1000 + b$

$b = 1000$

$C(x) = 0.2x + 1000$

fixed costs: \$1000

marginal cost: \$0.20/battery

3) The Hell's Angels are planning their annual fund-raising drive. They plan to sell cookies, and will charge $1.50 per cookie. The only expenses they will incur are the cost of the cookie dough, estimated at 25¢ per cookie, and the $400 charge for renting a booth at the local Renaissance Fair.

a) Write down the associated cost, revenue, and profit functions.

$C(x) = 0.25x + 400$

$R(x) = 1.50x$

$P(x) = 1.50x - (0.25x + 400) = 1.25x - 400$

b) How many cookies must the Hell's Angels sell in order to break even?

$1.25x - 400 = 0 \rightarrow 1.25x = 400$

$x = 320$

c) What profit (or loss) results from the sale of 400 cookies?

$P(320) = 1.25(400) - 400 = \$100$

4) You have a restaurant and you find that the daily demand for your enchilada plate is given by

$$q = -10x + 100$$

where q is the number of enchiladas you can sell in one day if you charge $\$x$ per enchilada. Ingredients amount to \$0.80 per enchilada to make them, and it costs \$10 per day to rent the restaurant. Find the daily cost as a function of the unit price x. Hence, find the daily profit as a function of x and determine the unit price you should charge to obtain the largest possible daily profit. What is the largest possible daily profit?

$C = 0.8q + 10 = 0.8(-10x + 100) + 10 = -8x + 90$

$R = xq = x(-10x + 100) = -10x^2 + 100x$

$P(x) = -10x^2 + 100x - (-8x + 90) = -10x^2 + 108x - 90$

Vertex: $x = \frac{-108}{2(-10)} = 5.4$

$P(5.4) = 201.6$

max profit \$201.60

5) A bacteria culture starts with 10,000 bacteria. Three hours later there are 11,000 bacteria. Find an exponential model for the size of the culture as a function of time t in hours, and use the model to predict how many bacteria there will be after 24 hours.

$$Q(t) = Q_0 e^{kt} = 10000e^{kt}$$

$$11000 = 10000e^{k(3)} \rightarrow \frac{11}{10} = e^{3k} \rightarrow \ln\left(\frac{11}{10}\right) = 3k$$

$$k = \frac{\ln(11/10)}{3} \approx 0.03177$$

$$Q(t) = 10000e^{0.03177t}, \quad Q(24) = 10000e^{0.03177(24)} = 21,436$$

6) Tom has just received won a lottery worth $500,000. He decides to take the annuity option that guarantees equal monthly payments over 20 years. The annuity earns 5.2% interest, compounded monthly. How much will his payments need to be so that the $500,000 draws down to zero after 20 years?

$$PMT = PV\frac{i}{1-(1+i)^{-n}} = 500000\frac{\frac{0.052}{12}}{1-\left(1+\frac{0.052}{12}\right)^{-(12)(20)}}$$

$$= 3355.27$$

7) The half-life of plutonium-238 is 88 years.

a) Obtain an exponential model for plutonium-238 in the form $Q(t) = Q_0 e^{-kt}$. (Round coefficients to three significant digits.)

$$k = \frac{\ln(\frac{1}{2})}{88} = -0.007877$$

$$Q(t) = Q_0 e^{-0.007877t}$$

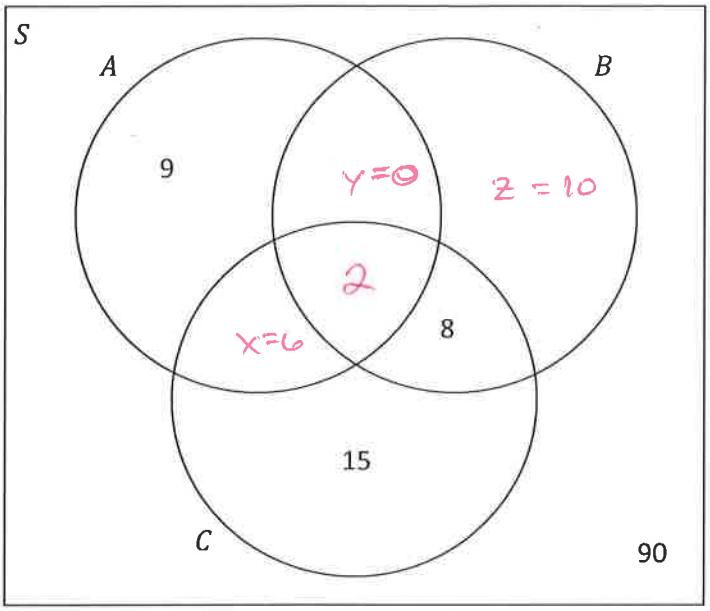
b) Use your model to predict, to the nearest year, the time it takes for one-third of the sample of plutonium-238 to decay.

$$\frac{2}{3}Q_0 = Q_0 e^{-0.007877t}$$

$$\ln\left(\frac{2}{3}\right) = -0.007877t$$

$$t = \frac{\ln(2/3)}{-0.007877} = 51.48 \text{ years.}$$

8) Use the given information to complete the solution of the partially solved Venn diagram.



$n(A) = 17, n(B) = 20, n(B \cap C) = 10, n(S) = 140$

$y + z + 2 + 8 = 20$

$y + z + 10 = 20$

$y + z = 10$

$x + y + 11 = 17$

$x + y = 6$

$x + y + z + 124 = 140$

$x + y + z = 16$

$6 + z = 16$

$z = 10$

9) A test requires that you answer first Part A and then either Part B or Part C. Part A consists of seven true-false questions, Part B consists of four multiple-choice questions with one correct answer out of five, and Part C consists of three questions with one correct answer out of six. How many different completed answer sheets are possible?

$2^7 \cdot 5^4 + 2^7 \cdot 6^3 = 107648$

10) Sven is given a bag containing 5 red marbles, 4 green ones, 3 white ones, and 2 purple ones. He grabs six of them. Find the probabilities of the following events, expressing each as a fraction in lowest terms.

$n(S) = C(14,6) = 3003$

a) He has all the red ones.

$n(E) = C(5,5) \cdot C(9,1) = 9 \quad P(E) = \frac{9}{3003} = \frac{3}{1001} \quad (\simeq 0.002 = 0.2\%)$

b) He has at least two white ones.

$n(E) = C(3,2) \cdot C(11,4) + C(3,3) \cdot C(11,3) = 990 + 165 = 1155$

$P(E) = \frac{1155}{3003} = \frac{5}{13} \quad (\simeq 0.385 = 38.5\%)$

c) He has three green ones and one of each of the other colors.

$n(E) = C(4,3) \cdot C(5,1) \cdot C(3,1) \cdot C(2,1) = 120$

$P(E) = \frac{120}{3003} = \frac{40}{1001} \quad (\simeq 0.04 = 4\%)$

d) He has no red ones.

$C(9,6) = 84$

$P(E) = \frac{84}{3003} = \frac{4}{143} \quad (\simeq 0.028 = 2.8\%)$